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Office of Sponsored Research
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14. ABSTRACT
We have completed a short program of theoretical research on dimensional reduction and approximation of models based on quantum stochastic differential equations. Our primary results lie in the area of modeling systems with time delays. We have developed a technique to approximate the dynamics of a linear passive QSDE network with time delays, within given limits on the complex plane, by truncation of an expansion in scattering poles.

15. SUBJECT TERMS
quantum probability, quantum stochastic differential equations

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UU	UU	UU	UU		Hideo Mabuchi
					19b. TELEPHONE NUMBER 650-723-0201

Report Title

Final Report: Approximation of Quantum Stochastic Differential Equations for Input-Output Model Reduction

ABSTRACT

We have completed a short program of theoretical research on dimensional reduction and approximation of models based on quantum stochastic differential equations. Our primary results lie in the area of modeling systems with time delays. We have developed a technique to approximate the dynamics of a linear passive QSDE network with time delays, within given limits on the complex plane, by truncation of an expansion in scattering poles.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received		Paper
02/25/2016	4.00	Yeong Dae Kwon, Michael Armen, Hideo Mabuchi. Femtojoule-Scale All-Optical Latching and Modulation via Cavity Nonlinear Optics, PHYSICAL REVIEW Letters, (11 2013): 203002. doi:
TOTAL:		1

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received	Paper
TOTAL:	

Number of Papers published in non peer-reviewed journals:

(c) Presentations

"Algorithms and software for quantum engineering," H. Mabuchi and R. Balu, Review Management Board meeting for the Advanced High Performance Computing Research Center, Stanford, CA, 13 Jan 2016

"Coherent feedback in photonic signal processing," H. Mabuchi, System-X Annual Review Meeting, Stanford, CA 19 Nov 2015

"Quantum dynamical networks and architectures," H. Mabuchi, ARO/ARL Workshop on Theoretical Foundations for Modeling of Open Quantum Systems, Adelphi, MD, 3 Aug 2015

"Coherent feedback in photonic signal processing and computation," H. Mabuchi, International Workshop on Principles and Applications of Control in Quantum Systems, Sydney, Australia, 21 Jul 2015

"Quantum information science and engineering," H. Mabuchi, seminar at Qualcomm, Santa Clara, CA, 4 May 2015

"Quantum engineering and coherent feedback," H. Mabuchi, seminar at Google, Venice, CA, 4 Feb 2015

"Architectures for autonomous quantum memories - coherent feedback and the small volume limit," H. Mabuchi, IARPA Workshop on Quantum Noise and Model Reduction, Laurel, MD, 26 Jan 2015

"Coherent feedback in quantum photonic circuit theory," H. Mabuchi, Optics and Electronics Seminar, Stanford, CA 5 Jan 2015

"Quantum information science and engineering," H. Mabuchi, seminar at Amazon, Seattle, WA, 20 Nov 2014

"Quantum photonic engineering: From single-device experiments to large-scale circuit architectures," H. Mabuchi, seminar at ARL, Adelphi, MD, 6 Nov 2014

"Quantum nonlinear optics and the renaissance of photonic computing," H. Mabuchi, physics colloquium at USF, San Francisco, CA, 29 Oct 2014

"Coherent feedback in quantum optical networks," H. Mabuchi, SECANT advisory board meeting, Albuquerque, NM, 9 Sep 2014

"Coherent feedback control of photonic circuits," H. Mabuchi, DARPA/MTO Meso Program Review, Monterrey, CA, 19 Aug 2014

"Coherent feedback and photonic engineering," H. Mabuchi, SPIE, Baltimore, MD, 7 May 2014

"Coherent feedback and quantum photonic circuit theory," H. Mabuchi, physics colloquium at Univ. of Utah, Salt Lake City, NV, 14 Nov 2013

"Physics with photons: from quantum to bio," H. Mabuchi, Statistics seminar, Stanford, CA, 5 Nov 2013

"Coherent feedback and quantum photonic circuit theory," Stanford-USTC-MIT Conference at USTC, Hefei, China, 17 Sep 2013

"Embedded control for smart photonics," H. Mabuchi, AFOSR Future Directions in Control, Arlington, VA, 19 Jun 2013

"Coherent feedback control and quantum photonic circuit theory," H. Mabuchi, Coherent Quantum Dynamics Summer School, Okinawa, Japan, 8 May 2013

Number of Presentations: 19.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
02/25/2016	2.00	Gil Tabak, Hideo Mabuchi. Trapped Modes in Linear Quantum Stochastic Networks with Delays, EPJ Quantum Technology (10 2015)
10/10/2013	1.00	Hideo Mabuchi, Dmitri S. Pavlichin. Photonic circuits for iterative decoding of a class of low-density parity check codes, ArXiv e-prints (06 2013)

TOTAL: **2**

Number of Manuscripts:

Books

Received	Book

TOTAL:

Received	Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Graduate Students

NAME	PERCENT_SUPPORTED	Discipline
Dmitri Pavlichin	0.50	
Jie Wu	0.13	
Ryan Hamerly	0.00	
Nikolas Tezak	0.00	
Gil Tabak	0.00	
FTE Equivalent:	**0.63**	
Total Number:	**5**	

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Dmitri Pavlichin	0.50
FTE Equivalent:	**0.50**
Total Number:	**1**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED	National Academy Member
Hideo Mabuchi	0.09	No
FTE Equivalent:	**0.09**	
Total Number:	**1**	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME	
Dmitri Pavlichin	
Total Number:	**1**

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

During the initial months of the award period we focused our efforts on the development and implementation of new computational methods for simulation (without approximation) of QSDE models that incorporate strong quantum noise and component nonlinearities, but minimal buildup of large-scale coherences. We applied this new method in a study of photonic circuit architectures for decoding low-density parity check codes, which have novel and important features. The results were reported in manuscript #1 (uploaded), which has been published in New Journal of Physics.

We then turned to the development of QSDE model reduction techniques for approximate simulation of large-scale optical computing networks. Our approach was to use QSDEs to construct a master equation model of the network, then to transform the master equation into a Wigner Equation framework, and finally to apply systematic Gaussian approximations to the PDE that governs time evolution. By doing so we obtained a set of coupled stochastic differential equations that can reproduce the circuit behavior up to leading order in quantum nonlinear effects. We extended this approach to
include not only quantum nonlinear optical models based on ideal (instantaneous, Kerr-type) nonlinearities, but also to models for free
carrier-based nonlinearities in optical semiconductors. The resulting sets of coupled SDEs include both photonic and electronic quantum fluctuations, and we are working to formulate near-term-feasible experimental tests for these models. In collaboration with Charles Santori at HP Labs, we applied the new model-reduction approach to a latch-based model of a 4-bit optical binary logic ripple counter. This enabled us to understand the impact of quantum fluctuations on circuit performance and to identify a critical switching-energy scale for this architecture of about 30 photons, below which quantum stochastic errors propagate and spoil the computation but above which the computation can be performed robustly.

In the final year of the award period we have focused on the challenge of modeling QSDE networks with time delays. The difficulty of accurately modeling dynamical systems with time delays is well known from classical theory; time delays technically add infinite-dimensional PDE-like degrees of freedom to a network model and must be treated carefully in dimensional reduction. In order to make progress on the quantum analogue we began with linear passive models representing the optical "skeleton" of the network, with the physical intuition (which still needs to be proved mathematically) that nonlinear interactions could be added to the reduced skeleton in a straightforward fashion. Building upon results from the classical literature in electromagnetic scattering and circuit theory, we showed how to derive a Blaschke-Potapov expansion for linear stochastic QSDE models with time delays and feedback, and developed a systematic procedure for truncating the expansion to arrive at a finite-dimensional model capturing the effects of the time delays within a desired window of timescales. We encountered a subtlety related to pure-feedforward channels in QSDE networks with time delays, and related this to an entire term that factors out of the Blaschke-Potapov expansion. We developed an approach to computing this term via systematic transformation of the QSDE model. The results of this work are described in manuscript #2 (uploaded), which has very recently been accepted for publication in EPJ Quantum Technology. We believe that these results will be of critical importance for modeling distributed quantum information processing networks.

Technology Transfer

Our ideas about quantum circuit architecture and modeling of networks with time delays have been discussed at length with Dr. Rad Balu and his team at ARL/Adelphi. The time delay approximation procedure is being incorporated into our QNET software package, which is being ported to DOD/HPC platforms by the ARL/Adelphi team.